Sample 4: An excerpt from the collection of problems.

4. TRIANGLES

4.4-7 Congruence of triangles. Some properties of isosceles triangles. An exterior angle of a triangle. Relations between sides and angles in triangles. Comparative lengths of a straight segment and a broken line connecting two points.

1. Three rods are hinged at their endpoints as shown in Figure 1. Can a different triangle be made from them by joining them up in different order? Substantiate your answer.

Figure 1

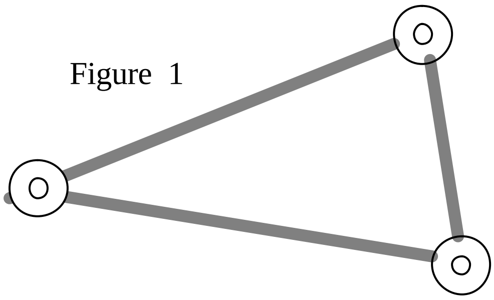
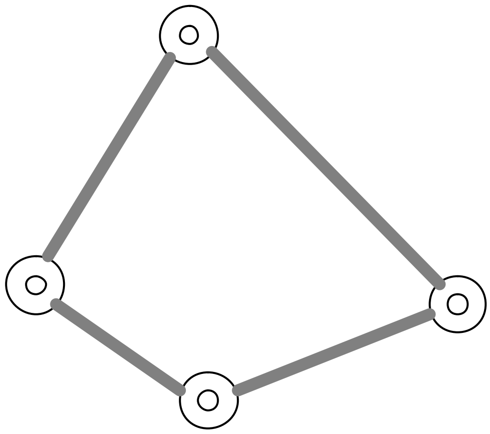


Figure 2



2. Four rods are hinged, as shown in Figure 2. Is the obtained figure rigid? If not, where would you put in the fifth rod to make the figure rigid? Substantiate your solution.

3. Figure 3 illustrates a method of determining the width of a river. Describe and substantiate the method.

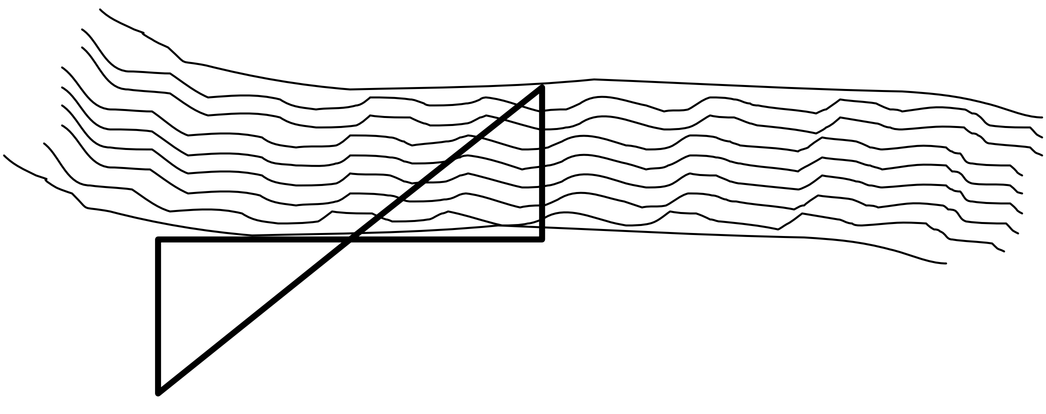


Figure 3

4. Prove that if a median of a triangle coincides with the altitude drawn from the same vertex, - then the triangle is isosceles.

5. Prove that if a bisector of an angle of some triangle coincides with the altitude drawn from the same vertex, - then the triangle is isosceles.

6. Prove that in an isosceles triangle a point in the altitude drawn to the base is equidistant from the vertices including the base.

7. Show that two perpendiculars to the same straight line cannot intersect. #

8. Point *M* is located in the interior of ΔABC. Which of the angles is greater: *ABC* or *AMC*?

9. Point *M* is located in the interior of ΔABC. The perimeter of which triangle is greater: *ABC* or *AMC*?

10. Prove that in an obtuse triangle, an altitude drawn from the vertex of an acute angle, cuts the extension of the opposite side, but not the side itself.#

11. Is it possible to cut a scalene triangle into two congruent triangles?

12. *AM* is a median in ΔABC. $AB>AC$. Which of the angles is greater: *AMB* or *AMC*? Substantiate.

13. * *AM* is a median in ΔABC. $AB>AC$. Which of the angles is greater: *BAM* or *CAM*? Substantiate.

14. Show that the foot of the altitude drawn to the greatest side of a triangle falls in the interior of the greatest side (not on its extension).

15. In ΔABC, $BC>AB>AC$. *AM* is the median and *AH* is the altitude drawn from the vertex *A*. Show that *H* is located between *C* and *M*. #

16. * *AM*, *AH*, and *AD* are, respectively, the median, the altitude, and the bisector drawn from the vertex *A* of ΔABC. Show that if $AB \neq AC$, point *D* is located between *M* and *H*. (In other words: a bisector of a triangle lies between the median and the altitude drawn from the same vertex).

17. Does a triangle with the sides equal, respectively, 4cm, 1cm, and 6 cm exist? Give reasons.

18. Prove that the sum of the segments connecting an interior point of a polygon with its vertices is greater than half-perimeter of the polygon.

19. Two triangles have a pair of common vertices and the third vertex of the one triangle lies within the interior of the other triangle. Show that the perimeter of the *included triangle* (triangle that lies inside the other one) is less than the perimeter of the *including triangle* (triangle that contains the other one in its interior). #

20. One triangle lies entirely within the interior of the other. Show that the perimeter of the included triangle is less than the perimeter of the including one. #

21. Is it true or false: If a polygon is *included* (lies in the interior) in some other polygon, the perimeter of the included polygon is the least of the two perimeters? – Support your answer. #

22. Prove: The perimeter of a convex polygon included in some other polygon is less than the perimeter of the including polygon.

23. Fermat's principle of optics states: A ray of light requires less time along its actual path than it would along any other path having the same endpoints. Based on that principle, prove that the ray of light traveling from *L* (light source) to *R* (receiver) in Figure 4 is reflected from the mirror m at point *A* such that the *angle of incidence,* a , equals to the *angle of reflection,* b .

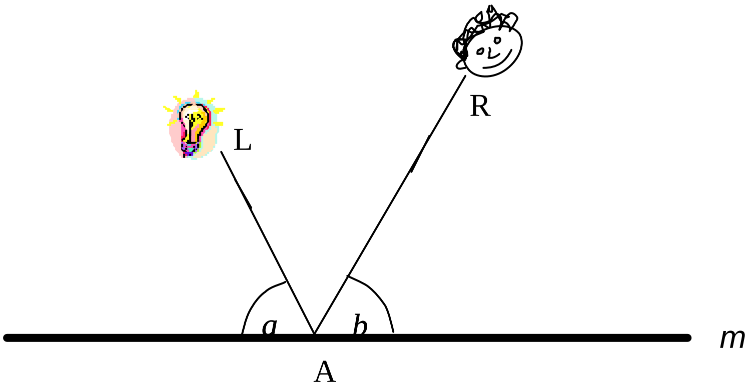


Figure 4

24. Prove that a median of a triangle is less than half-perimeter of the triangle.

25. Prove that a median of a triangle is less than half the sum of the sides that *include the median* (i.e. have a common vertex with the median) and greater than half the difference between the sum of the sides including the median and the third side.

26. It is known about the segments AB, AC, and BC : AB < AC+BC; AB > AC; AB > BC. Use RAA to prove that A, B, and C are not collinear.